Examiners' Report
Principal Examiner Feedback

Summer 2023

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2(WMF02) Paper 01

Examiners' Report for WFM02 Summer 2023

Overview

The paper this year provided a little bit challenge than some of recent years but still proved highly accessible throughout. Questions 1 and 4 were the most accessible, while question 2 was the most challenging by far, being the only question whose modal score was not full marks. The other questions all had a similar level of access, with question 3 and question 8(b) being and the last few marks of question 10 being the other discriminating parts of the paper at the top grades.

Question 1

A favoured topic and the most accessible question on the paper, with about 50% achieving the mode of full marks, and a further 30% scoring 5 or 6 marks. The method of differences method is well accustomed to being set and the question provided a good easing in to the paper at all grade levels.

Part (a) was the most challenging part of the question, with a significant number of students struggling with this part, though many of these found the rest of the question straightforward. Some tried to use a form of partial fractions and others tested particular values of r to show the two sides agreed at some values only. Those that crossed multiplied by the denominator of the left side and then used a sequence of equivalences, rarely scored both marks as their solution lacked a minimal conclusion. However, many were able to work a correct strategy through to completion.

In part (b) very few did not attempt to apply the method of differences (indeed, the question cited the method) and it was generally answered very well, showing sufficient steps to exemplify the cancellation. The students that did not reach the correct algebraic portion of the answer, had often not written out the terms for $r = n - 1$ and $r = n$ for cancelling correctly, or made a sign error when combining terms, with $\sqrt{n}-\sqrt{n-1}-1$ being common among incorrect answers. Some errors occurred in the constants, with inclusion of a "-2" terms sometimes seem. But overall the majority of students were successful in achieving the correct expression, and this part was the best answers of the three parts in this question.

Part (c) was again usually successfully answered, but less well so than part (b). Very few tried an incorrect attempt to evaluate f(50) − f(2) or f(50) − f(4), however, there were a significant number of attempts using the result of (a), rather than (b), thinking the $\sqrt{r}-\sqrt{r-2}$ was the function needed to evaluate the sum, while others just tried using the difference in the summand evaluated at 50 and 3, not appreciating the result just found in (b). A few started the question again with $n = 50$, repeating the differences method rather than using the result already found.

Algebraic slips in simplifying a correct expression were also regularly seen, losing the final accuracy mark.

Question 2

This proved to be a challenging question for many, with a wide mark distribution seen. It was the only question with a mode not full marks and mean score (~4.5/10) of less than half marks. The modal score was only 2/10 scored by 15% of students, with a fairly uniform spread across all the marks. Students at all grade levels struggled with aspects of this question.

Part (a) was based on standard techniques of simplifying/manipulating complex numbers in modulus-argument form, with most students able to successfully employ De Moivre's theorem at the start, achieving at least one mark for a correct application (usually on the numerator). The denominator proved more challenging for many, with not many dealing with the minus sign between terms before attempting to apply the theorem. However, it was acceptable for students to write the denominator as $\cos(\pi)-\mathrm{i}\sin(\pi)$ directly, as this arises from a fairly simple extension of De Moivre's theorem. Many went directly $\cos(-\pi)+\mathrm{i}\sin(-\pi)$, showing a better understanding of the theorem, but still lacking clear a clear step. Others attempted to move directly to exponential forms for the numerator and denominator, with varying levels of success for the denominator in particular, but again these were often not well explained. Another common occurrence was for students to go directly to $\cos(\pi)+\mathrm{i}\sin(\pi)$ in the denominator, and it is perhaps unfortunate that this happens to be equal to the correct denominator, due to $\sin(\pi)$ being zero, as it meant many reached the correct given answer through some erroneous work, which was not given credit. Students should, though, be encouraged to show all steps in a "show that" question.

The division technique was not well attempted by many, as the given solution was often arrived at without sufficient evidence. Success on this method often depended on the form of denominator found in the first steps, with the implication of "adding the arguments" for dividing complex numbers being often in evidence where a negative argument was reached. Working in exponential form was the most efficient method, but only a small percentage of students realised this, and even then adding the arguments was common. Those who did realise that subtraction was necessary and achieved an argument of $\frac{8\pi}{3}$ often went on to complete the part successfully. Some students simplified the denominator to equal -1, which made the "division" process much more straightforward, but often these did not show a sufficient step in sorting out the minus in the numerator before stating the given answer. Others attempted to multiply through numerator and denominator by the complex conjugate and the use trigonometric identities to arrive at the correct result. Again success was mixed in such approaches. It was rare to see attempts using Cartesian form, but one or two students managed to obtain the correct form from careful, if totally unnecessary work. It was interesting that most students could not achieve full marks on this question, which was based on standard techniques.

This part proved problematic for all, so was not useful as a discriminator as marks were as likely to be lost at A grade as at E grade.

Part (b) was more successful overall, with students here able to access at least two or three marks with relative ease, though full marks was rare. Almost all students recognised the form of the first of the given regions and obtained a circle. Obtaining two appropriate half-lines was less successful, though many did attempt a pair of lines. Use of the real axis as one line, or not having the lines meet at the centre of the circle, were the common errors with these. Shading the correct area, where both circle and a pair of lines were given, was better done, but students commonly lost the final mark even if the three methods were gained for not appreciating that the circle had to travel through the origin. Some students did not shade the region as requested, while others made their region ambiguous with careless shading.

Part (c) proved to be a better discriminator, with many students unsure how to progress and only those with a good understanding of the situation able to work out how to proceed. This part was often not even attempted, particularly by the lower grade students. Of those who were successful in this part, constructing an appropriate triangle was the most common approach, and applying the tangent ratio. Some students made an attempt for their diagram in (b) while others made wholly unacceptable attempts which did not tally with their diagram in (b) at all.

Question 3

This question provided some challenge and was a good discriminator for the paper. The modal mark of full marks was achieved by about 30%, with a further 20% dropping just one mark (usually for the inclusion of all the end points in part (b)), with 4 marks also being a common score. Most were able to make some progress in one part or the other, often restarting in part (b) where part (a) was not successfully completed.

Part (a) was a slight deviation from the norm in an inequalities question, asking for an intermediate one line inequality to be reached, rather than leaving it to the students to find the critical values by any algebraic means. Most attempted this by cross multiplication, and only relatively few students incorrectly cross multiplied, though it was not uncommon by way of error to do so. Most multiplied both sides by the square of the appropriate linear factors, usually including k^2, not realising k being positive meant the extra factor k was not needed. These often ended up with the extra k factor in their coefficients, so this did not affect the outcome. Some left the k in the denominator, which was equally fine. Only minority attempted to put terms over a common denominator, but this was a more successful approach when employed for the first mark, but was more problematic when it came to reaching the form shown in the question. Most were able to reach a correct answer, though there were many algebraic slips in working with the k's.

Many who struggled with part (a) went on to restart part (b) from the beginning to successfully achieve the critical values. Having a constant k seemed to confuse them compared with when k was equal to three.

In part (b) students that could not complete part (a) were still able to make progress by starting with 3 in the denominator to produce a fully correct solution. It was also common for students with a fully correct part (a) to restarted (b) again instead of substituting $k = 3$ into their previous result (and not always successfully). Whichever approach, most were able to produce 4 critical values from a correct method, though some did use incorrect methods to solve the quadratic, so scored no marks in this part. When students achieved four appropriate critical values from a correct method, they usually chose an appropriate pair of regions, but numerous cases of choosing the "outsides and middle region" were also noted, and these could score only the first M mark in this part. The mark scheme was perhaps a but more harsh than previous years, where a mark for the correct critical values may have preceded forming a correct region. However, even when a correct form of the region was selected with correct critical values, students often did not achieve the strict inequalities but included −4 and +1 in the regions to lose the final mark.

Question 4

This was the second most accessible question of the paper, following question 1. Nearly 45% achieved the modal full mark score, and 70% scoring 8 or more, with fewer than 10% scoring less than 3 marks. The solution of second order differential equations is a staple on this paper.

For part (a), nearly all students formed and solved the auxiliary equation correctly. The repeated root was dealt well by most but there were still a few students who had a complementary function of the form $(A+B)\mathrm{e}^{4x}$, not fully understanding of the concept of requiring two arbitrary constants.

Nearly all students used the correct form for the particular integral but some quadratics only had two terms instead of three, mimicking the form of the right hand side of the original equation missing an x term. Although most students had an excellent understanding of the process and methods required there was a lesser ability to carry out basic arithmetical calculations, with an incorrect constant term in the particular integral being the most common error in this question. Such a mistake here resulted in the loss of three accuracy marks in total and students would be well advised to check very carefully their working in these situations. Though less common that previous series,the final accuracy mark in part (a) was sometimes withheld because of students failure to put ‘’y =’’, instead having for example ‘’GS=’’. This was not penalised again in pat (b), when the particular solution was required, provided they had an ‘’equation’’, so the mistake was not costly.

Part (b) was again well answered by the majority of students. Those who wrote their complementary function in the expanded form of $A\mathrm{e}^{4x}+B\mathrm{e}^{4x}$ rather than $(A+B)\mathrm{e}^{4x}$ were slightly less likely to make mistakes when differentiating but there were many fully correct solutions to this question. The process was known by the majority, with slips in calculation occasionally losing a mark. However, there were a few students who did not understand what to do, who instead used the initial conditions to find the value of $\frac{d^2y}{dx^2}$ at $x = 0$ as if they were finding coefficients for a Taylor Series question. Usually they ended up accessing the first mark as well for setting up the first equation needed, but seldom went on to solve the question.

Part (c) posed little problem for anyone who advanced that far (though those who didn't complete (b) made no attempt). Substitution in to the equation to achieve an expression of the correct form was done by most, but occasionally only partial substitution was made. If a correct particular solution had been found a correct answer to (c) usually followed.

Question 5

This question proved to be one of the most successful questions involving complex transformations in recent years, possibly due to the aid of the given form of the circle equation. It was among the top four accessible questions on the paper with over 40% achieving full marks for it. Even when little progress was made in part (a) a mark or two was still available in part (b). Following full marks, 2, 6 or 3 out of 7 were the next most common scores.

In answering part (a) it was evident that most were well rehearsed in the art of rearranging to make z the subject of the equation, and then attempting to multiply by an appropriate conjugate to equate real and imaginary parts. The alternatives on the scheme were very rare to see, and certainly less successful, and no attempts via Apollonian form or use of points on the circle were recorded.

Common errors included sign errors when multiplying numerators or denominators, missing denominators when labelling real and imaginary parts, or other errors in processing rather than a lack of understanding. The final mark required fully correct work, and so was sometimes lost in otherwise correct solutions where the denominator was incorrect, or x and y associated only with the numerators of the u and v. It is also noteworthy that many were able to work out the correct value of k, for example by using the image a point on the line, or sometimes fortuitously from incorrect methods, or a correct method with errors, and these were still allowed access to part (b).

Part (b) was accessible for many, with at least the centre point being obtainable from the form of the circle equation given in the question. As noted, the correct value of k often arose even from incorrect work, and so those who did this and knew the method for finding the centre and radius of a circle from its Cartesian equation, Core 2 work, could score both marks. The centre was usually done well, with occasional sign slips, but there were often slips in algebra working out the radius.

Question 6

Although this was another highly accessible question, with 33% attaining the modal full marks and a further 20+% scoring at least 7 marks, it nevertheless ranked being only questions 2 and 8 for difficulty. Largely this was due to the dependency of marks meaning incorrect starts, or second steps, led automatically to the loss of later marks.

Part (a) of this question was routine for most further mathematicians, and familiarity with sec and tan was evident. Most students obtained a correct second derivative, but it was uncommon to see an identity used at this point to re-write their second derivative in terms of only sec. Most students then went on to use the product and chain rules successfully on their second derivative and proceeded to factorise first before using identities to obtain the required form. However, there were many who lost a power of a tan or sec at the third derivative stage (not applying the chain rule correctly or fully to their second derivative), who lost the second mark, and hence only scored 1/4 for this part. Sign or coefficient errors occasionally happened, but were less common. It was also rare, but not unknown, to see students working in terms of sin and cosine and such approaches usually led to errors in processing.

Part (b) was accessible for all with many students scoring at least one mark for finding values of their derivatives at the point where $x = \frac{\pi}{3}$. Substitution was not always seen for this, but usually the values were correct to imply the method - though students could be encouraged to make method clear in case of errors in processing. The second mark was for using a correct Taylor series, and most students did apply their values to a correct series, but some students made slips with substituting, and if they did not quote a correct Taylor's series formula then they risked losing the method mark here. Again, the importance of showing method should be stressed. Other common slips involved unsimplified coefficients, or incorrect values emanating from incorrect derivatives in (a), and a common error after scoring the first two marks from a correct third derivative was to lose the $\sqrt{3}$ in the $+\frac{23\sqrt{3}}{3}\left(x-\frac{\pi}{3}\right)^3$ term. Occasionally losses of one of the indices also occurred, but in this question there was no penalty for omitting the "$y =$", as has often been the case in the past, and may be the case again in the future.

Part (c) was generally successfully approached, and indeed should have been a routine process substituting $x = \frac{7\pi}{24}$ into their series and evaluating. Errors from part (b), or incorrect rounding, were the most common cause for a loss of mark here, with the method achieved by most. However, some students attempted to set $x - \frac{\pi}{3} = \frac{7\pi}{24}$, find x and then substitute this into the expansion, which yielded no marks, perhaps over thinking the question, or acting in anticipation without taking in what they question was actually asking for. A careful reading of questions is advised to ensure the correct question is being answered.

Question 7

Once again the modal mark was full marks, scored by 25% of students in this questions, which was surprising as the demand of the last few marks was considered high. So it was pleasing to see so many good responses. A further 12% managed 10 out of 11, with about 30% scoring either 7 or 8 marks.

In part (a) most students achieved a correct initial derivative and went on to form a correct expression linking $\frac{dy}{dx}$ and $\frac{dy}{dz}$ before substituting into equation (I), meaning full marks was common. Errors tended to be slips in algebra during the simplification process, though a few did make errors either in the initial derivative, or when using the chain rule, before forcing the result. There were also a few obscure attempts where students tried multiplying through or manipulating the original equation before substituting, with method not always being clear in such case. A good logical flow should be encouraged in "show that" questions. However the overriding method was shown to be understood by most, even if the execution of the method went awry through poor differentiation. Only very few tried to substitute into (II) to transform into (I).

For part (b) virtually all students recognised the correct method to solve this type of differential equation and many arrived at the correct simplified IF. A few did not incorporate the negative sign and integrated $+\frac{2}{x}$, or were unable to evaluate or simplify the integrating factor correctly. This lost the accuracy mark at this point of the question and also prevented access to the last 3 marks, where an attempt at an integral of the correct form was required. However, most did successfully reach the second method mark of the question for applying the integrating factor correctly. Many of these spent time multiplying throughout by the integrating factor to show the left hand side as $\frac{d}{dx}\left("x^{-2}"z\right)$ first, which was not necessary, and students well versed in the method were able to go directly to $I.F.z = \int I.F.\left(8x\ln x\right)dx$. It was at this point that the question proved a good discriminator for the paper with only a minority of students demonstrating a correct process to integrate $\frac{\ln x}{x}$ and access the last 3 marks, whether due to incorrect formation of the integral or simply not knowing how to carry out the integral of the correct form. Most who reached a suitable integral tried integration by parts but many could not work out how to deal with the second stage. A few spotted that it was a reverse chain rule and were able to just write the answer down. A few of the students who successfully proceeded this far missed the very last mark by forgetting the arbitrary constant, something that should be drilled as essential to the method in this type of question.

Question 8

This question was a fairly standard question on polar coordinates and students seemed to be well prepared for it. Though rated as the second hardest question on the paper, it performed significantly better than question 2, the hardest, and proved to be a good discriminator of students. Nearly 30% attained the modal full marks, though the next most common score was zero marks, but just over 10%, and a good spread of marks between was also scored. The zero scores were a mix of non-responses or attempts that were cut short and poor attempts that did not access marks. There was little indication that time was an issue overall.

Part (a) should have been routine, but there were some students (who made progress in the question) who failed to gain this first mark of this question. Of these the common error was to think that point A was where $r = 0$, and so give $\left(0, \pi\right)$ as the coordinate, but other misunderstandings were evident also.

Most students knew the method required for part (b) and only a very small number attempted to differentiate $r\cos\theta$ rather than $r\sin\theta$. This part of the question was better answered than in previous years with most students achieving a 3 term quadratic in $\cos\theta$ and solving successfully to achieve the correct coordinates. A common error was to end up with $r = 3$ instead of 9, though the angle was usually found correctly. Most showed sufficient working, but a small number of candidates worked out or guessed the angle to be $\frac{\pi}{3}$ either from no working or following a correct derivative, and such students locked themselves out of marks where no calculus approach was shown. Working must be shown to guarantee full marks.

The first four marks of part (c) was answered successfully by the majority of students, being a very common theme in polar curve equations. Students were obviously well prepared for a question of this type. There was a lot less “poor squaring” than in previous years, particularly missing of the middle term, though incorrect squaring of the 6 was not infrequent. The use of a double angle formula to integrate $\cos^2\theta$ was handled competently by the majority to achieve the correct integral.

The final four marks provided somewhat more of a challenge, as her the students needed to work out the correct approach for the problem in hand, whereas the first few marks could be scored by simply applying the correct area formula for polar curves. Even the substitution of the correct limits into the integral caused issues for some students, with limits $\frac{\pi}{3}$ to $\frac{\pi}{2}$ or even $\frac{2\pi}{3}$ seen regularly, stemming from incorrect attempts to decompose the area. Finding the height BQ was sometimes stumbled on by chance, without knowing what to do with it, but finding it as part of incorrect approached, and only the better students were able to find the area of the trapezium (or its equivalent), and the final, correctly. Students would be well advised to use a diagram (either a new or the existing one) to aid and explain their strategy when attempting to calculate the area of the trapezium, as a good decomposition on the diagram greatly helps. Indeed, the most successful attempts at this question tended to be those where the diagram in the question had been annotated to identify the relevant triangles and lengths required to gain traction with the problem.